Physics 200B Winter 2014

Problem Set IV: Not required, but strongly recommended for Final.

1.) Calculate the electrical conductivity of a system where electrons collide primarily with neutrals, with collision frequency ν_{en}. Assume $l_{mfp,e} < L_{macro}$, and use a Krook model.

2.) Show that a random walk in 3D will not return to its starting point, while a walk in 1D will.

3.) a) Starting from the Chapman-Kolmogorov Equation

$$f(\underline{p}, t+\Delta t) = \int d\underline{q}\; T(\underline{q}, \Delta t)\, f(\underline{p}-\underline{q}, t)$$

derive a Fokker-Planck equation in momentum.

b.) Show that this equation conserves number.

c.) Write general expressions for the drift $\underline{V}$ and diffusion D. Note these are on velocity space.

d.) Use the condition that the flux must vanish for an equilibrium distribution function to relate $\underline{V}$ and D.

e.) Make this real by taking f to be a distribution of a small number of heavy particles mixed with a gas of light particles. Consider the heavy particles as effectively stationary during a collision. Give an expression for the diffusion coefficient and drift velocity.

f.) How long will it take for a heavy particle to be slowed down by collisions with the lights?

g.) How does the radius of a beam of heavy particles evolve in time when passing thru the gas of lights?

4.) Use the krook model to calculate:
   a.) the shear viscosity
   b.) the thermal conductivity

of the usual monatomic gas. Explain clearly what these transport coefficients mean.

   c.) Calculate the rates of viscous relaxation and thermal relaxation will produce entropy.

5.) How does the Boltzmann H-theorem effectively determine the structure of the equilibrium distribution function?

6.) Estimate the mean free path of a gas of density n, v_{th} and force range d. Order the basic length scales.

7.) a.) What are the fundamental assumptions of Boltzmann's equation and theory of entropy?. What role do these play in the theory?
   b.) How is the BBGKY hierarchy truncated?.

8.) How would you reconcile:
   a.) the Poincare Recurrence theorem for Hamiltonian systems
with the
   b.) Boltzmann H-theorem and entropy production in a Hamiltonian system?

9.) How might you form an "H-theorem" for a Fokker-Planck theory with a drift? What would a state of zero entropy production correspond to?

10.) Describe the time structure of the process of relaxation to a state of maximal entropy. What are the scales, the type of relaxation, etc. Relate these to the Chapman-Enskog expansion.